15th Canadian Masonry Symposium
Ottawa, Canada
June 2-5, 2025

Structural Analysis of a Historic Masonry Rib Vault: A Case Study from Canada's Centre Block

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Abstract

Preserving our built heritage necessitates the analysis of historic structures under new loading conditions to bring structural performance up to modern codes and to accommodate changes to the building use or configuration. This case study looks at the performance of a masonry gothic rib vault ceiling under seismic loading and alteration to the existing load path.

The rib vault within Canada's central parliament building, Centre Block, was designed with traditional analysis methods, and has supported its self-weight for over a century. The rehabilitation of Centre Block involves the introduction of a movement joint that will seismically separate the attached Library of Parliament. This joint would disrupt the existing gravity load path of the historic rib vault ceiling requiring restraint of the thrust crossing the joint as well as a reassessment of the vault's stability.

Traditional analysis methods are adequate for assessing capacity and stability under gravity and seismic loads. However, these methods become insufficient when the gravity load path is altered, and finite element analysis (FEA) can provide useful insight into structural behaviour. This case study presents a methodology for validating an FEA model through traditional thrust line and kinematic methods. Modern standards are discussed and are applied to the results. This approach allows for the evaluation of the vaulted ceiling stability and informs the design of future interventions to ensure its preservation under new support conditions and seismic demands.

Keywords

Thrust line analysis, unreinforced masonry, finite element analysis, masonry arches, vaulted stone ceilings, seismic.

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INTRODUCTION

Historic stone masonry vaults and arches stand as enduring testaments to architectural ingenuity, skillfully combining functionality with aesthetic elegance. These structures, often found in our built heritage, face challenges when held to modern standards which typically occurs during building retrofits, renovations and rehabilitations. As such, the analysis of historic stone masonry arches is essential to understanding their structural behavior which is key to ensuring their preservation for future generations.

Assessing heritage vaults and arches poses unique challenges. This paper presents a case study of a heritage rib vault which must be structurally assessed as part of the rehabilitation of Canada's central parliament building. The masonry supports of the rib vault are being altered as part of the rehabilitation. In traditional methods there are guidelines on sizing buttresses and modern standards includes stiffness guidelines indicating where rigid supports are acceptable [1, 2]. In this alteration, the supports fall outside of these guidelines, this necessitates a finite element analysis (FEA) methodology to capture the changing support stiffness, going beyond the bounds of traditional analysis techniques. This FEA approach is validated using traditional analysis techniques and determines the stability of the vault based on modern standards.

This case study presents the thrust line analysis, kinematic analysis and FEA in the rib vault as-found condition, validating the FEA model for future use in designing and detailing the alterations to the vault supports.

CASE STUDY CONTEXT

Canada's central parliament building, Centre Block, is currently undergoing a significant rehabilitation. The building's structure is being analyzed and reinforced to ensure compliance to modern building code, the National Building Code of Canada (NBCC) 2020. To achieve this goal, the superstructure is being seismically isolated to reduce the seismic demand on the building and its heritage elements.

This case study focuses on the gothic ribbed vaulted stone ceiling that is located directly adjacent to the intersection of the Centre Block and the Library of Parliament buildings. The northern rib of the vaulted ceiling is supported by a masonry pier and buttress. The seismic isolation of Centre Block requires a movement joint between the Centre Block and the Library of Parliament, thus altering the masonry pier supports and altering the original load path. Preserving the rib vault is a high priority as it is the defining feature of Centre Block's Hall of Honour, the central corridor in the building, that functions as a key ceremonial space.

Figure 1: Hall of Honour

The objective of this case study is to analyze the structural stability of the rib vault by conducting both gravity and seismic analyses. Additionally, the study seeks to determine the horizontal forces (i.e. thrust) resisted by the adjacent Library of Parliament to design the movement joint detail between the two buildings.

As-found condition of vaulted ceiling

The rib vault, constructed in 1916, has no visible cracking and has a complete set of historic documents including detailing of the individual stone layout, arch radii, and stone thickness. This documentation was validated using point cloud scans from both the intrados (underside of ceiling) and extrados (top side of ceiling). While drawings record the arch stone fill thickness as 101 mm, scans indicate the actual built condition is closer to 152 mm when accounting for the uneven surface created by rough-cut stones and additional mortar on the extrados. Figure 2 depicts the vaulted ceiling geometry including nomenclature for ribs following principal curvature at each defined arch. The ribs forming arches in the vault are 355 mm thick as found in both historic documents and point clouds. The vault is comprised of Tyndall limestone masonry, which is still actively quarried and has a density of 2435 kg/m^3.

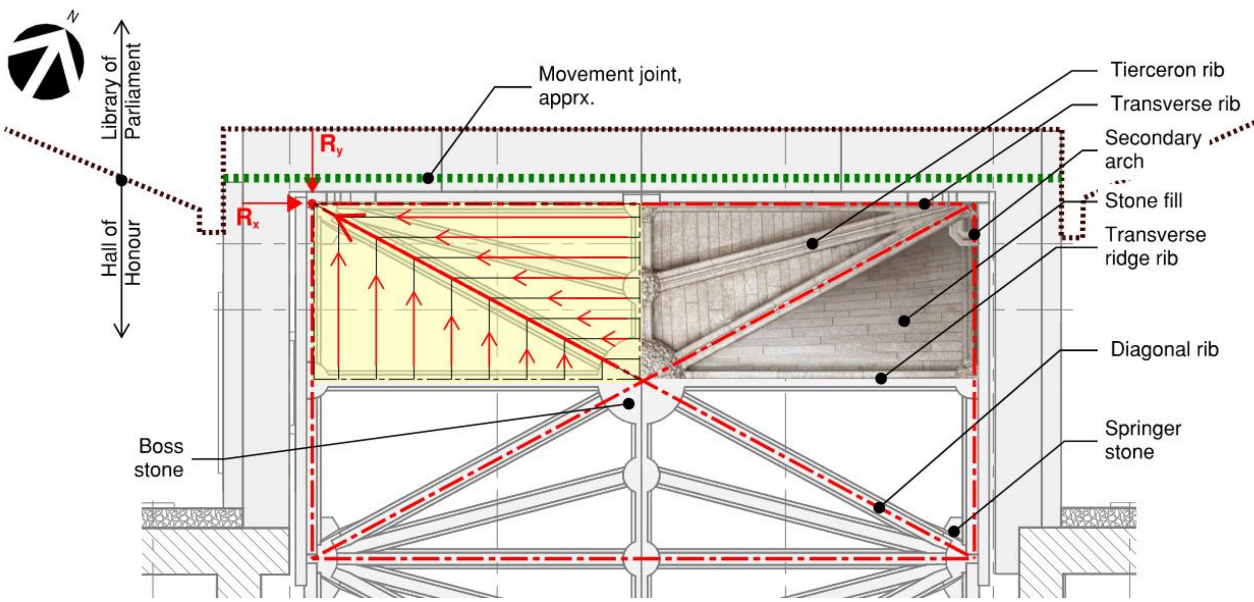

Figure 2: Vaulted ceiling plan

ANALYSIS

To evaluate the stability of the rib vault and determine the thrust exerted towards the library, this study employs a threefold analytical approach: graphic statics thrust line analysis under gravity loads, kinematic analysis under seismic loads, and FEA for both gravity and seismic scenarios.

The original rib vault, from 1916, was likely not designed to withstand seismic forces. The rib vault is required to resist a seismic acceleration equivalent to 0.067g, based on its location in Ottawa and considering the seismic isolation system. The seismic acceleration was determined from a non-linear time history analysis of the base isolated Centre Block model.

Graphic Statics Under Gravity Loads

Firstly, a graphical method was used to estimate the horizontal thrust forces due to self-weight at the support points of the Hall of Honour vaulted stone ceiling. The graphical thrust line analysis is consistent with methods employed at the time of original construction [1]; this uses force vectors proportioned to the arch self weight to solve the indeterminate arch. [3]. The vertical force vectors were determined by discretizing one symmetric quadrant of the ceiling into 450 mm wide 2-D slices in both orthogonal directions along the diagonal arch including the stone ribs, stone fill, and the boss stone. The graphic statics method determines horizontal reactions using the catenary principle; each discretized slice is related geometrically, where the vertical force vectors are applied at the center of gravity and forces are in equilibrium. Since the location of the actual thrust force within the thickness of the arch at the crown is unknown, two possible thrust line curves were derived using the vertical force vectors by solving for the minimum and maximum thrust forces as shown in Figure 3 below.

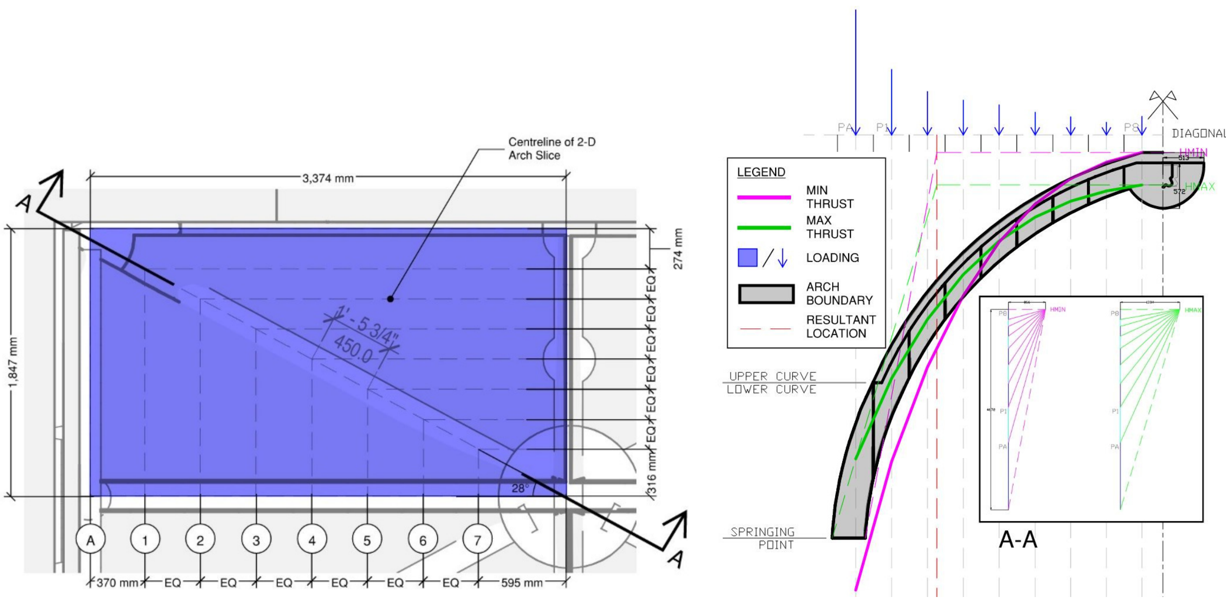

Figure 3: Discretized plan quadrant (left), and graphic statics along diagonal arch (right)

Equilibrium was achieved using a compression-only solution within the vaulted ceiling geometry, i.e., the masonry was assumed to have no tensile capacity, no sliding, and infinite compression strength ensuring no crushing could occur. Force vectors were calculated for each slice, with magnitudes equivalent to the weight of the geometric volume and located at the centroid, and the line of thrust was generated starting at the crown of the arch to the springing point. This method determined that the thrust force applied against the library wall was approximately 16% of the total weight of the vaulted ceiling, resulting from the maximum thrust line that falls within the thickness of the arch (Table 1).

Stability can be determined by finding a single thrust line that is fully contained within the geometric boundaries of the arch. As depicted in Figure 3, the minimum thrust line falls outside the boundary of the diagonal arch while the maximum thrust line is contained within this boundary. Given a solution for the diagonal arch supporting the full weight of each quadrant exists for the maximum thrust line, no intermediary thrust lines needed to be solved to deduce that the vaulted ceiling is stable under self-weight. It is worth noting that the results of the graphic statics analysis presented are a lower bound of the actual ultimate load that would cause failure [3]. Under gravity loads, to derive the upper bound limit, kinematically admissible mechanisms would need to be found iteratively by applying live loadings; however, given the vaulted ceiling is self-supporting only, and no cracks were observed on site, this additional analysis was not required.

The calculated thrust force is highly influenced by the arch geometry, stone fill thickness, chosen load path, and analysis method. Therefore, the magnitude of the thrust force against the library required further validation using a detailed FEA model generated from point cloud scans.

Kinematic Method Under Seismic Loads

The kinematic method was used to assess the capacity of the diagonal, transverse, and secondary arches under seismic loading to determine stability. This was achieved by determining the maximum horizontal acceleration at which a sufficient number of hinges will form causing instability. This method assumes that the masonry arch will not fail due to a crushing or sliding failure and that the masonry has no tensile strength [1].

Hinges can form in any location within the arch and each potential hinge combination must be considered to determine the minimum horizontal acceleration causing instability. The applied loads result in asymmetric loading in the arch, therefore four hinges will form to create global instability [1]. To determine the hinge location, the method as outlined by Dimitri & Tornabene [4] was followed and solved iteratively to find hinge locations and minimum lateral loads resisted by the 2D arch.

Using this method, the arches can resist the following seismic demands: 0.24g for the transverse arch, 0.45g for the secondary arch, and 0.14g for the diagonal arch. All three arch profiles were found to be stable, when considering the arch tributary width as uniform throughout; however, this does not match the 3D nature of the vault, where the mass is higher in the mid-span of the arch and lower towards the supports. This distribution suggests that the results of the kinematic analysis are unconservative.

Finite Element Analysis

The FEA model used to validate the thrust under gravity was refined for the purpose of detailed analysis. The SAP2000 model geometry was simplified such that the ribs are represented as frame elements and the stone fill is represented with equivalent line loads. The model extents align with the lowest springing line of the vaulted ceiling. Where the ribs converge onto a single stone block, they are constrained such that they displace in unison. Within this converged zone a weight modifier is applied to overlapping frame elements.

The vault supports are modeled as spring elements corresponding to the masonry pier geometry found in historic documentation, vertical supports are located at the vault base and horizontal supports are located at the vault base and where the ribs converge onto one stone block. The transverse ridge rib members are modeled for the purpose of transferring load between rib arches, but the results for these horizontal members are not included as their curvature is not captured and they notch into the stone fill such that they are uniformly supported. The frame elements were discretized into segments cut at uniform vertical intervals. A sensitivity analysis assessed intervals from 50mm to 800mm, when comparing the magnitude of support reactions and maximum internal forces under gravity loads, there was a minimal change in results, less than 1% difference, for intervals 200mm or less. The final analysis model has 100mm intervals, resulting in frame elements ranging from 100mm to 323mm in length.

This model includes two self-weight load cases to envelope the stone fill thickness as indicated in historic drawings and point cloud measurements.

Section Analysis Under Gravity Loads

The results of the FEA model under gravity loading, when compared to the findings of the thrust line analysis, help validate the modeling approach taken. The resulting thrust towards the library for the 100 mm thick stone fill is higher but generally aligns with the graphic statics thrust for both absolute thrust and the thrust-to-weight ratio, as shown in Table 1. As expected, the analysis for the 152 mm thick fill load case results in a higher absolute thrust into the library. The most conservative result will be used in the design of future alterations for the movement joint.

Table 1: Thrust towards library

	Graphic Statics	FEA – Gravity Analysis		FEA – Seismic Analysis	
Stone Fill Thickness (mm)	100	100	152	100	152
Thrust towards Library (kN)	7.7	9.6	12.3	12.7	15.5
Vault Quadrant Weight (kN)	46.8	48.8	62.9	48.8	62.9
Thrust-to-Weight Ratio	16%	20%		26%	25%

To determine the arch stability, results for each frame element in the vault under dead load is plotted showing the applied loads relative modern and traditional limits in Figure 4 and is visually depicted, where red elements indicate failure in Figure 5. The results from SAP2000 were reported as axial and bending forces and were converted to axial force and eccentricity. The maximum eccentricity of an uncracked section is the kern distance, equal to one sixth the section depth, which equates to the thrust line contained within the middle third of the stone section. When cracking is deemed acceptable, CSA S304 [2] allows for a maximum eccentricity equal to one third of the section depth, equating to a crack that extends 50% of the rib depth. For the stone ribs, only a few elements exceed the CSA S304 limit, with a 65% crack depth being the maximum; this would have been considered acceptable by traditional methods as the thrust line is contained within the depth of the arch. The location of the maximum eccentricity is at the apex of the secondary arch.

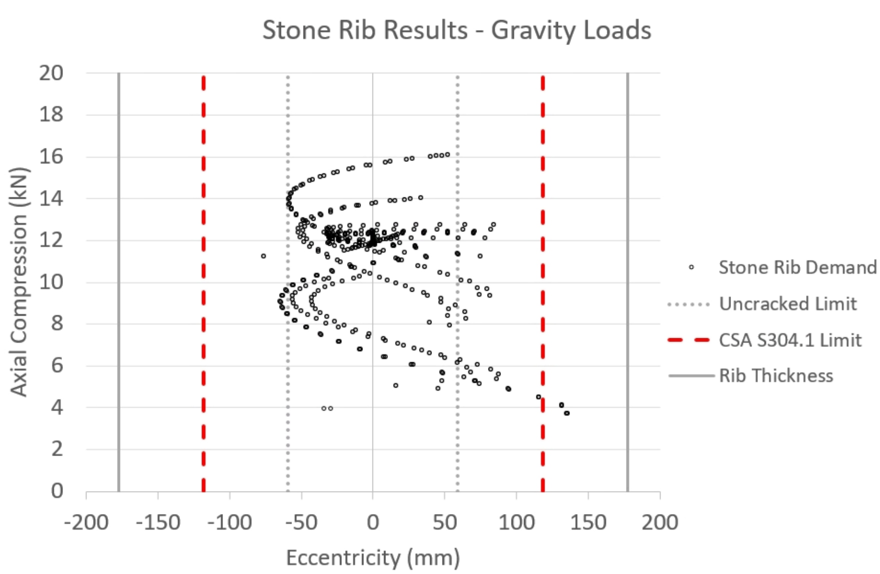

Figure 4: Arch frame elements under gravity loads with 100 mm stone fill.

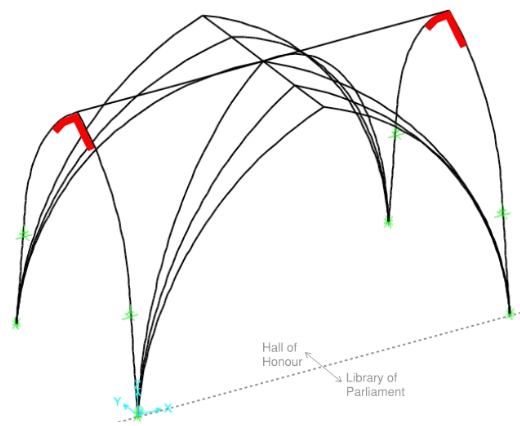

Figure 5: Vault ribs under gravity loads, red indicates CSA S304 limit is exceeded.

Section Analysis Under Seismic Loads

Seismic loads are applied to the FEA model as an equivalent static load of 0.067g, the results show many frame elements exceeding CSA S304 limits. The failing rib segments under seismic loads are shown in Figure 6 and Figure 7, where the seismic load is applied parallel to each arch span to envelope the problem. When the seismic load is applied parallel to the secondary arch, there are segments where the eccentricity of the axial load falls outside the thickness of the stone ribs. This indicates that under these loads the rib vault fails, with a crack opening through the full thickness of the arch, well beyond the traditional limit and the CSA S304 limit. When the seismic load is applied parallel to the diagonal rib and transverse rib, the eccentricity of the axial load falls outside the CSA S304 limit in multiple segments, indicating failure. The FEA results for the 3D analysis, subjected to a 0.067g seismic load identifies failure zones as shown in Figure 7. These are local failures as per the limits in CSA S304, which are unacceptable and will require structural intervention.

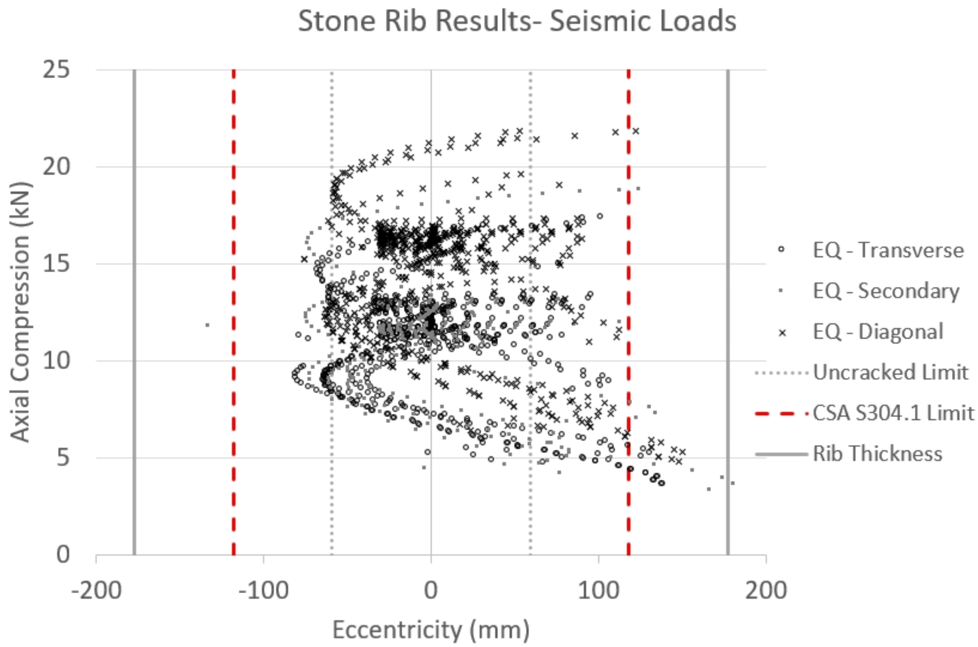

Figure 6: Arch frame elements under gravity and seismic loads (EQ) parallel to each main rib, for 100mm stone fill.

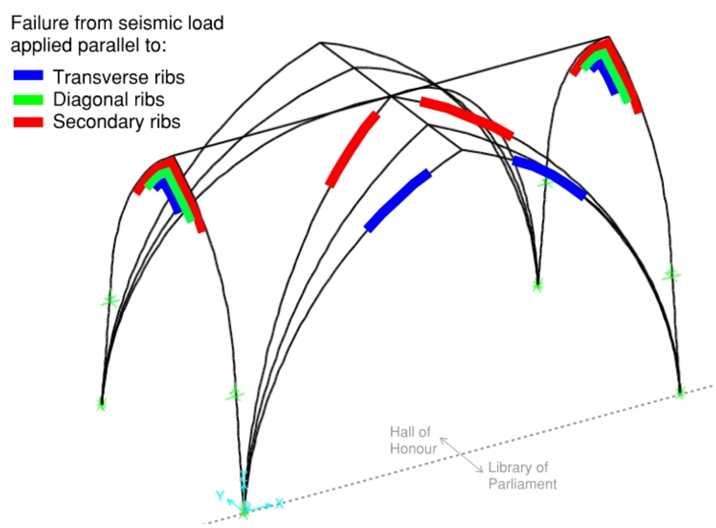

Figure 7: Vault ribs under gravity and seismic loads, failure to meet CSA S304 limit

When the FEA results were interpreted using the kinematic method failure criteria, there were fewer than four failure zones per arch, therefore there is local failure, but global stability is maintained, as there is no four-hinge collapse mechanism. Both the kinematic and FEA results agree that a four-hinge mechanism will not form at the applied 0.067g seismic force. The FEA analysis was then pushed to global instability to compare the locations of hinge formation and magnitude of seismic load applied parallel to each arch span, as summarized in Table 2. When pushed to four-hinge failure all ribs remain in axial compression.

The seismic force at which global instability is reached is different between the kinematic method and FEA results (Table 2). This can be attributed to affects of the 3D geometry, differences in the modelled support stiffness and the use of the more stringent CSA S304 failure criteria. To test the impact of these factors, an additional 2D FEA model is created, isolating the transverse arch geometry. This model includes only uniform self weight of the stone rib excluding the stone fill, has rigid supports and hinges are formed when the axial load falls outside the rib depth. Under these conditions, the onset of global instability for the 2D transverse arch occurs at 0.26g, much closer to the results from the kinematic method, thus validating the FEA approach. The location of the failure zones for each method are shown in Figure 8.

To determine the maximum thrust towards the library at the rib vault supports, seismic loads are applied northward towards the library. The inclusion of the seismic load results in greater thrust than found in the thrust line and gravity analysis methods and is compared in Table 1, this load case governs, with a thrust-to-weight ratio of 25%-26%.

Table 2: Summary of arch hinge formations under seismic loads parallel to span

	Transverse Arch Hinge Locations			Diagonal Arch Hinge Locations		Secondary Arch Hinge Locations	
Hinge	Kinematic Method	2D FEA Failure Zones	FEA Failure Zones	Kinematic Method	FEA Failure Zones	Kinematic Method	FEA Failure Zones
A	30	0°-1.6°	32°-47°	35	28°-47°	10°	0°-22°
B	80	32°-86°	59°-90°	80	54°-90°	80°	52°-90°
C	135	108°-147°	145°-148°	145	114°-125°	130°	107°-128°
D	180	178°-180°	178°-180°	180	147°-154°	180°	164°-180°
Force (*g)	0.24	0.26	0.33	0.14	0.27	0.45	0.27

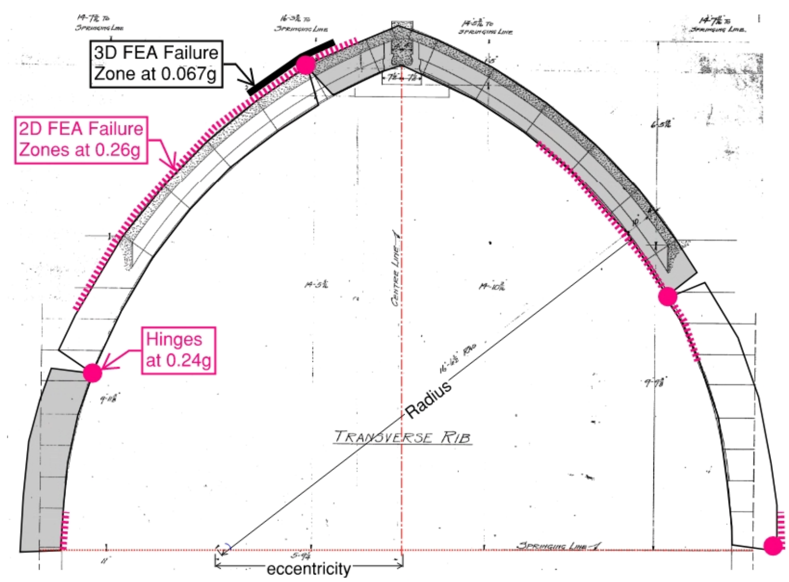

Figure 8: Hinge formations vs FEA failure zones along 2D transverse rib arch

DISCUSSION

Historic analysis techniques are often an appropriate method when analyzing stone masonry arches. The assumptions made using traditional methods must be carefully considered and modern standards must be followed.

This case study highlights the challenges of applying 2D traditional analysis methods to 3D geometries. It shows the differing results between 2D and 3D analyses, particularly in kinematic analysis, which assumes a 2D arch of uniform thickness and cannot capture multiple spring supports or interactions between connected ribs in the vault. Despite these differences, 2D kinematic analysis plays a crucial role in validating the FEA results.

The failure criteria outlined in CSA S304 are more stringent than traditional analysis methods regarding the eccentricity limits for unreinforced masonry. The standard also introduces a masonry compression limit, which traditional methods do not consider, but as found in this case study, compression failure does not typically govern when the arch supports only its self-weight. Both traditional and modern analyses assume that stone masonry resists zero tension.

Traditional analysis criteria limits thrust lines to within the thickness of the masonry elements. CSA S304 stipulates that the eccentricity of the compression force must be within the middle two-thirds of the arch, equivalent to a maximum crack depth of 50%. Traditional graphical thrust line analysis can easily be adapted to this limitation.

In this case study, the FEA analysis of the masonry vault under gravity load does not satisfy the CSA S304 limit, because the eccentric axial force falls outside the middle two-thirds. However, it does satisfy the limit from traditional methods of analysis and historically has not shown any signs of distress. If the scope of analysis was limited to gravity loads alone, and there were no changes to loading or construction, it would be reasonable to rely on traditional limits. Nevertheless, when seismic loads are introduced, the vault fails to satisfy modern criteria. This instability under seismic conditions necessitates a structural intervention to ensure the vault's safety and long-term stability.

Future Work

This case study highlights the complexity of analyzing masonry arches and shows that this arch is stable under gravity loads but fails under seismic loads, necessitating a structural intervention. The FEA model, validated in this paper using traditional analysis methods, will serve as a baseline for further analysis and intervention design.

Intervention is required both to support the vault under seismic loads and to accommodate the movement joint that will separate the Centre Block from the Library of Parliament. This movement joint and associated alteration to the masonry piers supporting the vault must effectively resist the rib vault thrust directed towards the library. The movement joint, will be incorporated into the FEA model to assess the changing load path and ensure the stability of the structure in its final condition.

CONCLUSIONS

The analysis of historic masonry vaults for seismic loading requires a balanced approach that integrates traditional and modern analysis methods. This case study on the Centre Block rib vault highlights the complexities of assessing such structures, especially for seismic retrofits.

By applying graphic statics, kinematic analysis, and finite element analysis, this study provides insight into the vault's structural behavior under both gravity and seismic loads and validates the results found in the

FEA model. The results determine that the vault fails under seismic loads, therefore an intervention is necessary. This intervention must be designed to preserve the structure's heritage integrity.

Modifications to the vault's supports will be required as part of the Centre Block's seismic isolation system. In anticipation of these changes, this study outlines the thrust forces transferring from Centre Block to the Library of Parliament, which will be structurally separated. Minimizing interventions to historic structures is always a priority in heritage conservation. In this case, the advantages of the base isolation system—reducing seismic demand on the Centre Block structure, including the rib vault—far outweighs the impact on the vault's support stiffness. This is particularly true in this case since the rib vault fails in its as-found condition under seismic loads and requires an intervention regardless.

This paper documents the steps taken to validate the model and lays the groundwork for designing structural interventions that will preserve the rib vault's architectural and historical integrity. A holistic approach to the detailing and design of the vault intervention will require the expertise of an interdisciplinary team to ensure a heritage-sensitive solution, preserving this character-defining feature of national significance.

ACKNOWLEDGEMENTS

The authors wish to acknowledge the support of our client, Public Services and Procurement Canada (PSPC), in publishing this work.

REFERENCES

[1] P. Roca, P. B. Lourenço and A. Gaetani, Historic Construction and Conservation: Materials, Systems and Damage, Taylor & Francis, 2019
[2] Canadian Standards Association (CSA), "S304 Design of masonry structures," 2024.
[3] J. Heyman, The Stone Skeleton. Structural Engineering of Masonry Architecture., Cambridge: Cambridge University Press, 1995
[4] R. Dimitri and F. Tornabene, "A parametric investigation of the seismic capacity for masonry arches and portals of different shapes," *Engineering Failure Analysis 52,* pp. 1-34, 2015.