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SENSITIVITY ANALYSIS OF MASONRY STRUCTURES

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ABSTRACT

A key point related to the reliability of numerical analysis is to know the importance of the material parameters and its influence on the structural response. This paper shows the results from a sensitivity study of masonry structures with respect to the material parameters. Both, micro-modeling, in which units and joints are represented separately, and macro-modeling, in which units and joints are represented as a continuum, are considered. The analysis concentrates on masonry shear walls which, traditionally, have been the most used configuration to test masonry structures. It is shown that, for practical purposes, the numerical analysis of masonry structures, is only slightly sensitive to reasonable variations in the material data.

INTRODUCTION

Masonry is a composite material made of units and mortar joints, being the interface between these materials essential for the behavior of the composite. A detailed analysis of masonry, hereby denoted *micro-modeling*, must then include a representation of units, mortar and the unit/mortar interface. A different approach can be used, hereby denoted *macro-modeling*, where the material is regarded as an anisotropic composite and a relation is established between average masonry strains and average masonry stresses. As one is dealing with three different components (units, mortar and interface), a lot of material parameters are necessary to characterize masonry. Even if masonry is considered as a homogeneous continuum, one is still dealing with a complex anisotropic material and the number of needed parameters are also high.

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The number of parameters necessary for material models has largely increased in the last decades, with the increase of sophistication of the models. It is now widely accepted that the post-peak behavior of the material must be included in the laws describing the material behavior. Recently, powerful and modern material models have been proposed by the author for micro-modeling [Lourenço and Rots 1997] and macro-modeling [Lourenço *et al.* 1997]. In the present paper, the sensitivity of these models is evaluated with the analysis of masonry shear-walls.

SENSITIVITY OF MICRO-MODELING

A composite interface model, which includes a tension cut-off for mode I failure, a Coulomb friction envelope for mode II failure and a cap mode for compressive failure, has been proposed for a simplified modelling strategy of masonry structures [Lourenço and Rots 1997], see Figure 1. Complete material data for this model can be obtained from recent experimental programs, see e.g. [Binda *et al.* 1988, Atkinson *et al.* 1989, Atkinson and Yan 1990, Van der Pluijm 1993, Van der Pluijm 1997].

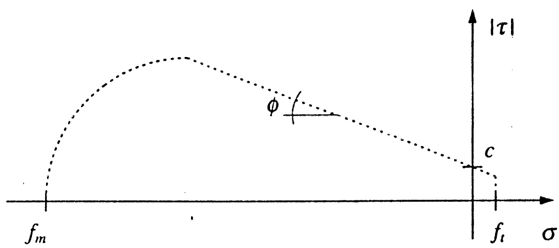


Figure 1 - Interface model for masonry joints [Lourenço and Rots 1997]

Interface elements permit discontinuities in the displacement field and their behaviour is described in terms of a relation between tractions { t } and relative displacements { Δu } across the interface. The linear elastic relation between these generalized stresses and strains can be written in the standard form as

$$\{\sigma\} = [D]\{\varepsilon\} \tag{1}$$

where { σ } = { t } = (σ , τ)$^{\mathrm{T}}$, [D] = $diag$ (k_n , k_s) and { ε } = { Δu } = (Δu_n , Δu_s)$^{\mathrm{T}}$, with n and s denoting the normal and shear components, respectively. Here, σ is the normal stress, τ is the shear stress and k is the stiffness.

The inelastic material parameters of the model are the following: f_t is the tensile strength of the joint, G_f^I is the mode I fracture energy of the joint, c is the cohesion of the joint, G_f^{II} is the mode II fracture energy of the joint, tanϕ measures the friction of the joint, tanψ measures the dilatancy of the joint (uplift of one unit over the other upon shearing), f_m is the compressive strength of masonry, G_f^c is the compressive fracture energy and C_s measures the contribution of the shear stress to compressive failure. It is noted that fracture energy is the area under the stress-crack_displacement diagram in a direct tension test, which measures the amount of energy necessary to open a crack of unitary area. This definition is used similarly for shear and compressive behavior.

The influence of the parameters $\tan\psi$ and C_s has been studied in Lourenço [1996] and will not be repeated here. In particular, for a range of reasonable values, it has been shown that the influence of the C_s factor is insignificant and the influence of $\tan\psi$ is very significant and critical for the analysis. The values recommended, $C_s = 9$ and $\tan\psi = 0$, were adopted in the analysis carried out in the present study.

Next, the sensitivity of the analysis with respect to the other material parameters will be studied. Two different groups were made: the first group includes the elastic properties (represented by k_n and k_s), the tensile strength f_t, the cohesion c, the tangent of the friction angle $\tan\phi$ and the masonry compressive strength f_m; the second group includes the different fracture energies, in tension G_f^I, in shear G_f^{II} and in compression G_f^c. For the first group, it is assumed that a close estimation of each material parameter is possible. Therefore, the values of the original analysis will be multiplied and divided by a factor 1.25 (yielding a variation 0.8 ÷ 1.25 of the original value), see Table 1. For the second group, it is assumed that a close estimation of each parameter is difficult, because much less experimental data are available. Therefore, the values of the original analysis will be multiplied and divided by a factor 2.0 (yielding a variation 0.5 ÷ 2.0 of the original value), see Table 2.

Table 1 - Better known parameters, for which extensive experimental data are available (analysis with 0.8 ÷ 1.25 of the original value)

k_n and k_s	f_t	c	$\tan\phi$	f_m

Table 2 - Worse known parameters, for which scarce experimental data are available (analysis with 0.5 ÷ 2.0 of the original value)

G_f^I	G_f^{II}	G_f^c

The example adopted for the micro-modeling study is a shear wall with an opening for which a complete discussion of results has been given by Lourenço and Rots [1997]. The complex failure behavior of the wall triggers all modes of the model but, in the end, a compressive dominant failure will be encountered. The numerical load - displacement diagram and the numerical results at peak load are shown in Figures 2 and 3. The experimental behavior is described by Raijmakers and Vermeltfoort [1992]. The central opening defines two small relatively weak piers and forces the compressive strut that develops under horizontal loading to spread around both sides of the opening. The collapse mechanism is formed with failure of the compressed toes, located at the bottom and top of the wall and at the bottom and top of the small piers. Four cracks remain active and the wall behaves similarly to four rigid blocks connected by hinges.

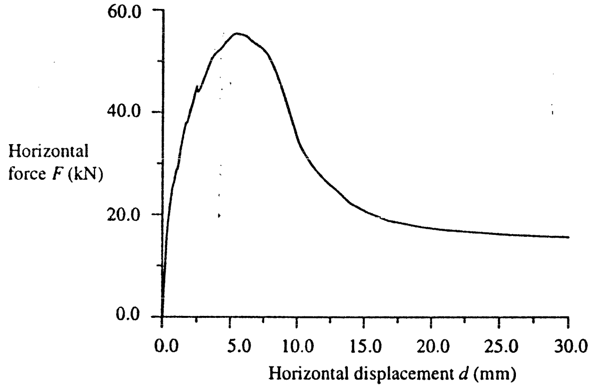


Figure 2 - Force-displacement diagram for masonry shear wall (micro-model)

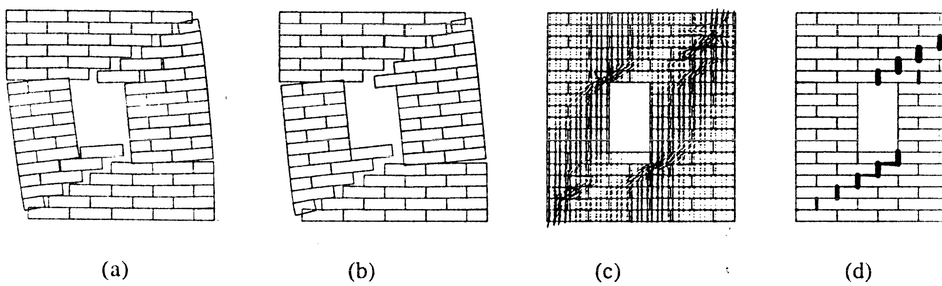


Figure 3 - Masonry shear wall (micro-model). Results of the analysis at peak load: (a,b) total and incremental deformed meshes; (c) principal stresses; (d) interface opening

The analysis has been repeated for different material properties according to Table 1 and Table 2. The results are shown in Figure 4.

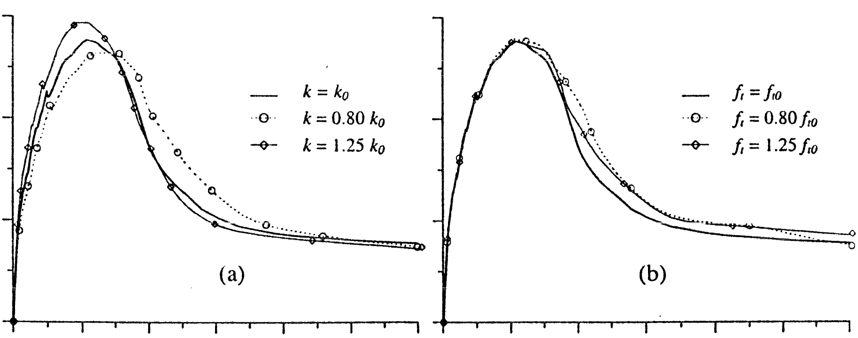


Figure 4 - Influence of the material parameters on the force-displacement diagram for masonry shear wall (micro-model): (a) elastic stiffness; (b) tensile strength (cont.)

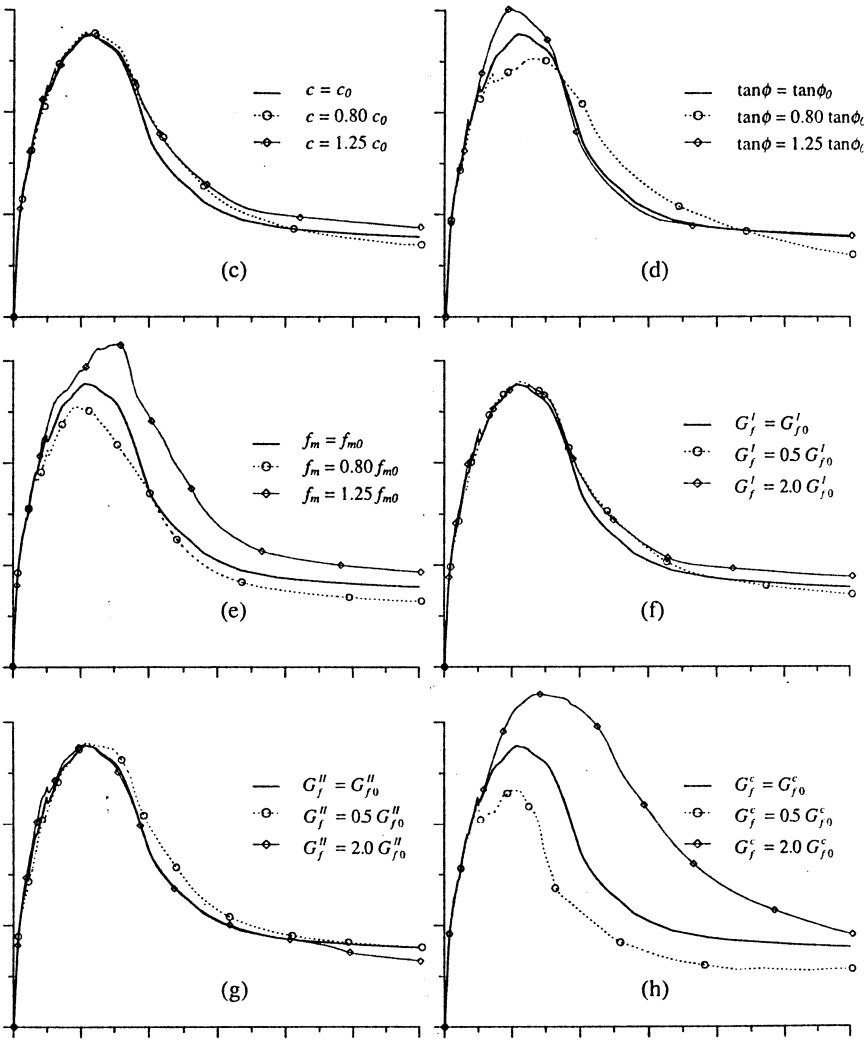


Figure 4 - Influence of the material parameters on the force-displacement diagram for masonry shear wall (micro-model): (c) cohesion; (d) friction angle; (e) compressive strength; (f,g,h) mode I, mode II and compressive fracture energy (contd.)

A comparison in terms of collapse loads is given in Table 3 and Table 4. The first table shows the results for the material parameters which are better known and the second table shows the results for the material parameters which are worse known.

Table 3 - Collapse loads obtained in the sensitivity analysis of a shear wall (values in kN). First series of parameters (micro-model)

Parameter	k	f_t	c	$tan\phi$	f_m
Divided by 1.25	52.81	55.92	55.70	50.50	50.79
Multiplied by 1.25	58.80	55.47	55.08	60.46	63.17
Difference to original	6.2%	1.1%	0.6%	9.6%	14.1%

Table 4 - Collapse loads obtained in the sensitivity analysis of a shear wall (values in kN). Second series of parameters (micro-model)

Parameter	G_f^I	G_f^{II}	G_f^c
Divided by 2.0	55.88	56.16	46.68
Multiplied by 2.0	55.19	55.54	65.54
Difference to original	0.9%	1.4%	18.6%

The conclusions from the results of this analysis are summarized in the following:

- It was observed that slightly different failure mechanisms were obtained with similar collapse loads. This indicates that numerical and experimental failure patterns may differ because the randomness of real material data cannot be directly reproduced;
- The results are almost insensitive to the variation of tensile strength, cohesion, mode I and mode II fracture energy. This is interesting for masonry as, for unconfined masonry, the failure mechanism is normally determined by tensile and shear behavior;
- The analysis is not insensitive to the variation of the parameters. In particular, the results are *slightly* sensitive to the stiffness of the joints, *moderately* sensitive to the friction angle and sensitive to the compressive strength and compressive fracture energy. Nevertheless, it must be stressed that the results appear to be *stable*, in the mathematical sense that a variation in the parameters is not propagated by the analysis.

SENSITIVITY OF MACRO-MODELING

A composite anisotropic continuum model, which includes a Rankine-*like* tension criterion and a Hill-*like* compressive criterion, has been proposed for a macro-modelling strategy of masonry structures [Lourenço *et al.* 1997], see Figure 5. Material data for the model can be obtained from experimental programs of biaxial loading of masonry, see e.g. [Page 1981, Ganz and Thürlimann 1982, Page 1983, Guggisberg and Thürlimann 1987, Lurati *et al.* 1990]

The Hooke´s law for continuum elements can be written in the standard form as

$$\{\sigma\} = [D]\{\varepsilon\} \tag{2}$$

where the elastic stiffness matrix $[D]$ can be obtained from four elastic constants (two Young´s moduli, one Poisson ratio and one shear modulus): E_x, E_y, ν_{xy} and G_{xy}.

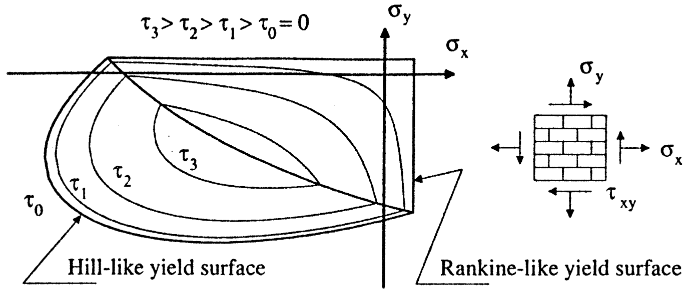


Figure 5 - Anisotropic material model for continuum elements [Lourenço *et al.* 1997]

The inelastic material parameters of the model are the following: f_{tx} and f_{ty} are the tensile strengths, G_{fx} and G_{fy} are the tensile fracture energies, f_{mx} and f_{my} are the compressive strengths, G_{fcx} and G_{fcy} are the compressive fracture energies, α is a parameter which controls the contribution of the shear stress to tensile failure, β is a parameter which controls the coupling of normal stresses with regard to compressive failure and γ is a parameter which controls the contribution of the shear stress to compressive failure. In the above, the subscripts x and y refer to the correspondent material axis.

Next, the sensitivity of the analysis with respect to these material parameters will be studied. Again, two different groups were made: the first group includes the elastic properties (represented by E_x, E_y, ν_{xy} and G_{xy}), the tensile strengths f_{tx} and f_{ty}, the compressive strengths f_{mx} and f_{my}, the parameter α and the parameter β; the second group includes the tensile fracture energies G_{fx} and G_{fy}, the compressive fracture energies G_{fcx} and G_{fcy}, and the material parameter γ. For the first group, it is assumed that a close estimation of each parameter is possible. Therefore, the values of the original analysis will be multiplied and divided by a factor 1.25, see Table 5. For the second group, it is assumed that a close estimation of each parameter is difficult because much less experimental data are available or higher variance of experimental data has been found. Therefore, the values of the original analysis will be multiplied and divided by a factor 2.0, see Table 6.

Table 5 - Better known parameters, for which lower variance has been found (analysis with 0.8 ÷ 1.25 of the original value)

E_x, E_y, ν_{xy} and G_{xy}	f_{tx}	f_{ty}	f_{mx}	f_{my}	α	β

Table 6 - Worse known parameters, for which higher variance has been found (analysis with 0.5 ÷ 2.0 of the original value)

γ	G_{fx}	G_{fy}	G_{fcx}	G_{fcy}

The example adopted is a masonry shear wall without an opening analyzed by Lourenço [1996] and built of 18 courses of masonry, from which the top and bottom courses are fully clamped in steel beams. The diagram for the horizontal force F vs. the horizontal displacement of the top boundary plate d is given in Figure 6. The behavior of the wall at peak load is depicted in Figure 7 in terms of total deformed meshes, incremental deformed meshes, cracked and crushed Gauss points. Masonry crushing is represented by triangles, with a size proportional to the compressive equivalent plastic strain. Initially, two horizontal cracks develop at the top and bottom of the wall. Upon increasing deformation, a diagonal crack arises. The diagonal crack progresses in the direction of the compressed corners of the wall, accompanied by crushing of the toes until total degradation of strength in the compressive strut.

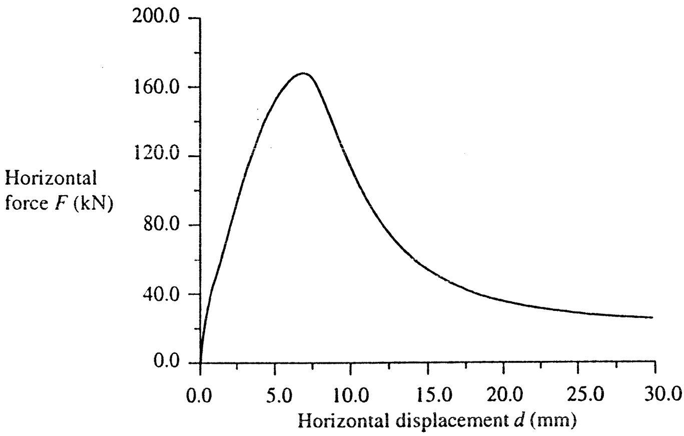
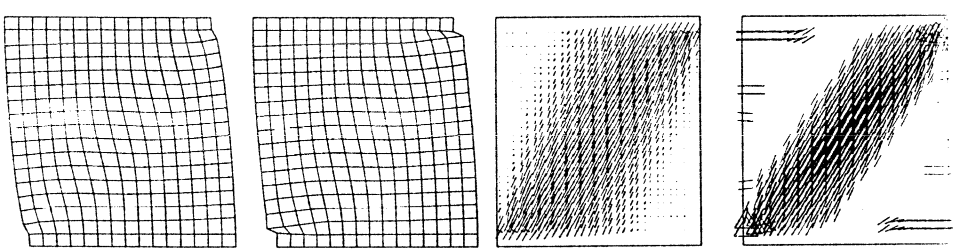


Figure 6 - Force-displacement diagram for masonry shear wall (macro-model)



(a) (b) (c) (d)

Figure 7 - Masonry shear wall (macro-model). Results of the analysis at peak load: (a,b) total and incremental deformed meshes; (c) principal stresses; (d) cracks and crushed points

The analysis has been repeated for different material properties according to Table 5 and Table 6. The results in terms of load-displacement diagrams are shown in Figure 8. In all the analysis, there were no changes in the failure mode discussed above and illustrated in Figure 7.

It can be observed that, within the limits established, the results are practically insensitive to the variation of the following parameters: elastic properties (represented by E_x, E_y, ν_{xy} and G_{xy}), the tensile strengths f_{tx} and f_{ty}, the compressive strengths f_{mx} and the parameter β. The results are sensitive to: the compressive strength f_{my}, the parameter α and the parameter γ, in terms of peak load; the compressive fracture energy G_{fcx}, in terms of post-peak stiffness; the compressive fracture energy G_{fcy}, in terms of peak load and post-peak stiffness.

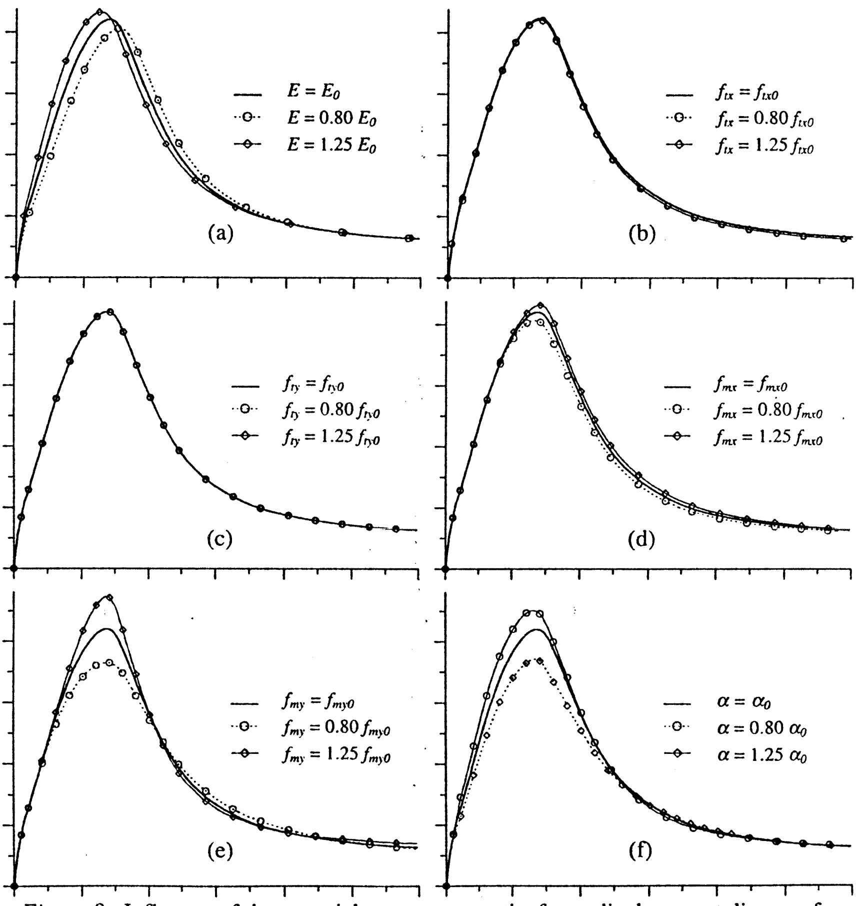


Figure 8 - Influence of the material parameters on the force-displacement diagram for masonry shear wall (macro-model): (a) elastic stiffness; (b,c) tensile strength in the x and y direction; (d,e) compressive strength in the x and y direction; (f) parameter α (cont.)

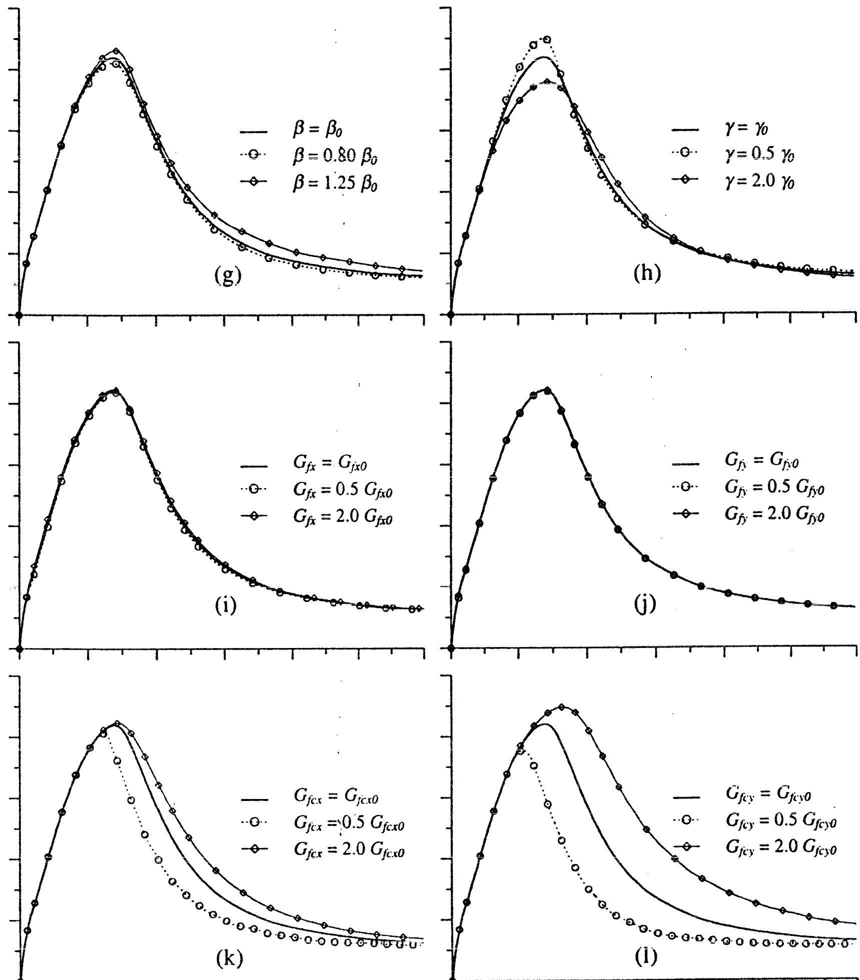


Figure 8 - Influence of the material parameters on the force-displacement diagram for masonry shear wall (macro-model): (g,h) parameters β, γ, (i,j) tensile fracture energy in the x and y direction; (k,l) compressive fracture energy in the x and y direction (contd.)

A comparison in terms of collapse loads is given in Table 7 and Table 8. The first table shows the results for the material parameters which show lower variance and the second table shows the results for the material parameters which show higher variance.

Table 7 - Collapse loads obtained in the sensitivity analysis of a shear wall (values in kN). First series of parameters (macro-model)

Parameter	E	f_{tx}	f_{ty}	f_{mx}	f_{my}	α	β
Divided by 1.25	162.4	167.8	168.0	162.8	146.0	180.5	164.6
Multiplied by 1.25	172.9	168.3	168.0	172.7	189.4	148.5	172.7
Difference to original	3.4%	0.2%	0.1%	3.2%	13.1%	11.7%	2.7%

Table 8 - Collapse loads obtained in the sensitivity analysis of a shear wall (values in kN). Second series of parameters (macro-model)

Parameter	γ	G_{fx}	G_{fy}	G_{fcx}	G_{fcy}
Divided by 2.0	180.3	167.5	167.8	162.5	151.1
Multiplied by 2.0	151.8	169.3	168.4	169.5	179.0
Difference to original	9.7%	0.7%	0.2%	3.3%	10.1%

The conclusions from the results of this analysis are summarized in the following:

- The results are almost insensitive to the variation of tensile strength and tensile fracture energy. This is quite interesting for masonry as, in most cases of unconfined masonry structures, the failure mechanism is determined by the tensile behavior;
- The analysis is not insensitive to the variation of the parameters. In particular, the results are sensitive to the compressive strength in the y direction, to the α parameter, to the γ parameter and the compressive fracture energy in the y direction. Again, it must be stressed that the results appear to be *stable*, in the mathematical sense that a variation in the parameters is not propagated by the analysis. The fact that an error in α parameter, which is a part of the tensile characterization of the composite behavior of masonry in tension, is only divided by a factor two is quite relevant.

CONCLUSIONS

Due to the large number of material parameters involved in models, for such a complicated material as masonry, and the scarce knowledge about the material, analyses have been carried out to assess the influence of the variation of the material parameters on the structural response. Both, micro and macro-modeling have been considered.

Practical structures have been selected for analysis and reasonable variations in the material parameters have been attributed. It has been shown that the response is sensitive to the variation of the parameters but the largest difference encountered in terms of peak-loads was smaller than 20%. This is reasonably high but might still be found acceptable for practice, in case of non-linear analysis. Nevertheless, an important issue is that the variation in the structural response is stable, in the sense that is not propagated by the analysis. Another important point is that the largest differences were found for compressive dominated failures. In the case of tensile / shear dominated failures, which are the typical cases of unconfined masonry structures, the maximum difference found was only about 10%.

ACKNOWLEDGMENTS

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Atkinson, R.H., Yan, G.G. 1990. Results of a statistical study of masonry deformability, TMS Journal, 9(1), pp. 81-94.

Atkinson, R.H., Amadei, B.P., Saeb, S., Sture, S. 1989. Response of masonry bed joints in direct shear, J. Struc. Engrg., ASCE, 115(9), pp. 2276-2296.

Binda, L., Fontana, A., Frigerio, G. 1988. Mechanical behavior of brick masonries derived from unit and mortar characteristics, in: Proc. 8th IB2MAC., Elsevier Applied Science, London, UK, pp. 205-216.

Ganz, H.R., Thürlimann, B. 1982. Tests on the biaxial strength of masonry (in German), Report No. 7502-3, Inst. of Struc. Engrg., ETH Zurich, Switzerland.

Guggisberg, R., Thürlimann, B. 1987. Experimental determination of masonry strength parameters (in German), Report No. 7502-5, Inst. of Struc. Engrg., ETH Zurich, Switzerland.

Lourenço, P.B. 1996. Computational strategies for masonry structures, Dissertation, Delft University of Technology, Delft University Press, The Netherlands.

Lourenço, P.B., Rots, J.G. 1997. A multi-surface interface model for the analysis of masonry structures, J. Engrg. Mech., ASCE, 123(7), pp. 660-668.

Lourenço, P.B., de Borst, R., Rots, J.G. 1997. A plane stress softening plasticity model for orthotropic materials, Int. J. Numer. Meth. Engng., 40, pp. 4033-4057.

Lurati, F., Thürlimann, B. 1990. Tests in concrete masonry walls (in German). Report No. 8401-3, Inst. of Struc. Engrg., ETH Zurich, Switzerland.

Page, A.W. 1981. The biaxial compressive strength of brick masonry, Proc. Intsn. Civ. Engrs., Part 2, 71, pp. 893-906.

Page, A.W. 1983. The strength of brick masonry under biaxial compression-tension, Int. J. Masonry Constr., 3(1), pp. 26-31.

Raijmakers, T.M.J., Vermeltfoort, A.T. 1992. Deformation controlled tests in masonry shear walls (in Dutch), Report B-92-1156, TNO-Bouw, Delft, The Netherlands.

Van der Pluijm, R. 1993. Shear behavior of bed joints, in: Proc. 6th North American Masonry Conf., Philadelphia, Pennsylvania, USA, pp. 125-136.

Van der Pluijm, R. 1997. Non-linear behavior of masonry under tension, Heron, 42(1), pp. 25-54.